Chapter 9 Exercise 9.1

Q. 1. (i) 25

(ii) 8

(iii) 9

(iv) -343

(v) 1

(vi) 1

Q. 2. (i) 4^3

$= (2^2)^3$

$= 2^6$

(ii) $(-2)^{20}$

$= 2^{20}$ (even power)

$= 2^{20}$

(iii) 6^{19}

(iv) $(-3)^3$

$= -3^3$ (odd power)

(v) $(-5)^{12}$

$= 5^{12}$ (even power)

(vi) $-(-1)^{100}$

$= -(1^{100})$

$= -1^{100}$

(vii) $-(7)^3$

$= -7^3$

(viii) $-(-11)^3$

$= -(-11^3)$ (odd power)

$= 11^3$

(ix) $(-1)^6$

$= 1^6$ (even power)

$= 1^6$

Q. 3. $-(-4)^7 = (4)^7$?

$= -(-4^7)$ (odd power)

$= 4^7$

Q. 4. (i) $(7.8)^3$

$= 474.55$

(ii) $\frac{6.5^5}{2.3^2}$

$= 2{,}193.37$

(iii) $(2.5^3)^5$

$= 931{,}322.57$

(iv) 12.45^0

$= 1$

(v) $2.5^3 \times 3.8^4$

$= 3{,}258.03$

(vi) $((5.1)^2)^3 \times (2.8)^2$

$= 137{,}954.9$

(vii) $\frac{(6.2)^2}{(5.8)^3}$

$= 0.2$

(viii) $(2.4)^3 \times (3.7)^2 \times (5.2)^5$

$= 719{,}538.26$

Q. 5.

Index notation	2	2^2	2^3	2^4	2^5	2^6	2^7
Whole number	2	4	8	16	32	64	128

Index notation	3	3^2	3^3	3^4	3^5	3^6
Whole number	3	9	27	81	243	729

Index notation	4	4^2	4^3	4^4	4^5	4^6
Whole number	4	16	64	246	1,024	4,096

Index notation	9	9^2	9^3	9^4	9^5	9^6
Whole number	9	81	729	6,561	59,049	531,441

(a) $x = 2$ or $x = 4$ or $x = 6$
$y = 1$ $y = 2$ $y = 3$

(b) $x = 1$ or $x = 2$ or $x = 3$
$y = 2$ $y = 4$ $y = 6$

(c) $x = 1$ or $x = 3$ or $x = 5$ or $x = 7$
$y = 1$ $y = 2$ $y = 3$ $y = 4$

(d) $x = 1$ or $x = 2$ or $x = 3$
$y = 1$ $y = 2$ $y = 3$

Q. 6. (i) 5^5 (iv) 4^8
(ii) 8^{10} (v) 2^{21}
(iii) 6^3 (vi) 3^7

Q. 7. (i) $(-5)^{10}$
$= 5^{10}$ (even power)
(ii) $(-2)^9$
$= -2^9$ (odd power)
(iii) $(-3)^7$
$= -3^7$ (odd power)
(iv) $(-5)^5$
$= -5^5$ (odd power)
(v) $(-7)^8$
$= 7^8$ (even power)
(vi) $(-4)^5$
$= -4^5$ (odd power)

Q. 8. (i) 3^1
(ii) 2^6
(iii) 10^3
(iv) 7^7
(v) 10^6
(vi) 8^{-6}

Q. 9. (i) 3^{15}
(ii) 6^{20}
(iii) 10^{25}
(iv) 4^{30}
(v) 7^{42}
(vi) 5^{36}

Q. 10.

Index notation	5	5^2	5^3	5^4	5^5	5^6
Whole number	5	25	125	625	3,125	15,625

(i) 25×125
$= 5^2 \times 5^3$
$= 5^5$

(ii) $3{,}125 \times 15{,}625$
$= 5^5 \times 5^6$
$= 5^{11}$

(iii) 625×25
$= 5^4 \times 5^2$
$= 5^6$

(iv) $(15{,}625)^{11}$
$= (5^6)^{11}$
$= 5^{66}$

(v) $25^3 \times 625^5$
$= (5^2)^3 \times (5^4)^5$
$= 5^6 \times 5^{20}$
$= 5^{26}$

(vi) $\dfrac{(15{,}625)^4}{(125)^3}$
$= \dfrac{(5^6)^4}{(5^3)^3}$
$= \dfrac{5^{24}}{5^9}$
$= 5^{15}$

Q. 11. (i) 2^6

(ii) 4^3

(iii) 64^1

(iv) $4{,}096^{\frac{1}{2}}$

There are only 4 other ways.

Exercise 9.2

Q. 1. (i) 3^{-3}

$= \frac{1}{3^3}$

$= \frac{1}{27}$

(ii) 5^{-2}

$= \frac{1}{5^2}$

$= \frac{1}{25}$

(iii) 9^{-2}

$= \frac{1}{9^2}$

$= \frac{1}{81}$

(iv) 7^{-3}

$= \frac{1}{7^3}$

$= \frac{1}{343}$

(v) 4^{-1}

$= \frac{1}{4^1}$

$= \frac{1}{4}$

(vi) 2^{-5}

$= \frac{1}{2^5}$

$= \frac{1}{32}$

Q. 2. (i) $6(4^{-3})$

$= 6\left(\frac{1}{4^3}\right)$

$= \frac{6}{64}$

$= \frac{3}{32}$

(ii) $2(7^{-2})$

$= 2\left(\frac{1}{7^2}\right)$

$= \frac{2}{49}$

(iii) $6(3^{-2})$

$= 6\left(\frac{1}{3^2}\right)$

$= \frac{6}{9}$

$= \frac{2}{3}$

(iv) $5(2^{-4})$

$= 5\left(\frac{1}{2^4}\right)$

$= \frac{5}{16}$

(v) $2(8^{-2})$

$= 2\left(\frac{1}{8^2}\right)$

$= \frac{2}{64}$

$= \frac{1}{32}$

(vi) $3(6^{-2})$

$= 3\left(\frac{1}{6^2}\right)$

$= \frac{3}{36}$

$= \frac{1}{12}$

Q. 3. (i) $\sqrt{100}$

$= 10$

(ii) $\sqrt{144}$

$= 12$

(iii) $\sqrt[3]{27}$
$= 3$

(iv) $\sqrt[4]{16}$
$= 2$

(v) $\sqrt[5]{32}$
$= 2$

(vi) $\sqrt[10]{1}$
$= 1$

(vii) $\sqrt[3]{1{,}000}$
$= 10$

(viii) $\sqrt[4]{81}$
$= 3$

(ix) $\sqrt[6]{64}$
$= 2$

(x) $\sqrt[3]{125}$
$= 5$

Q. 4.

x	1	2	3	4	5	6	7	8	9	10
x^2	1	4	9	16	25	36	49	64	81	100

x	1	4	9	16	25	36	49	64	81	100
$\sqrt{x}$	1	2	3	4	5	6	7	8	9	10

Q. 5.

x	1	2	3	4	5	6	7	8	9	10
x^3	1	8	27	64	125	216	343	512	729	1,000

x	1	8	27	64	125	216	343	512	729	1,000
$\sqrt[3]{x}$	1	2	3	4	5	6	7	8	9	10

Q. 6. (i) $100^{\frac{1}{2}}$
$= \sqrt{100}$
$= 10$

(ii) $64^{\frac{1}{2}}$
$= \sqrt{64}$
$= 8$

(iii) $216^{\frac{1}{3}}$
$= \sqrt[3]{216}$
$= 6$

(iv) $512^{\frac{1}{3}}$
$= \sqrt[3]{512}$
$= 8$

(v) $16^{\frac{1}{2}}$
$= \sqrt{16}$
$= 4$

(vi) $8^{\frac{1}{3}}$
$= \sqrt[3]{8}$
$= 2$

(vii) $9^{\frac{1}{2}}$
$= \sqrt{9}$
$= 3$

(viii) $1{,}000^{\frac{1}{3}}$
$= \sqrt[3]{1{,}000}$
$= 10$

(ix) $64^{\frac{1}{3}}$
$= \sqrt[3]{64}$
$= 4$

(x) $36^{\frac{1}{2}}$
$= \sqrt{36}$
$= 6$

Q. 7. (i) $16^{\frac{1}{4}}$

$= \sqrt[4]{16}$

$= 2$

(ii) $27^{\frac{2}{3}}$

$= \left(\sqrt[3]{27}\right)^2$

$= 9$

(iii) $64^{\frac{2}{3}}$

$= \left(\sqrt[3]{64}\right)^2$

$= 16$

(iv) $16^{\frac{3}{4}}$

$= \left(\sqrt[4]{16}\right)^3$

$= 8$

(v) $100^{\frac{3}{2}}$

$= \left(\sqrt{100}\right)^3$

$= 1{,}000$

(vi) $125^{\frac{2}{3}}$

$= \left(\sqrt[3]{125}\right)^2$

$= 25$

(vii) $16^{\frac{5}{4}}$

$= \left(\sqrt[4]{16}\right)^5$

$= 32$

(viii) $81^{\frac{3}{4}}$

$= \left(\sqrt[4]{81}\right)^3$

$= 27$

(ix) $9^{\frac{3}{2}}$

$= \left(\sqrt[2]{9}\right)^3$

$= 27$

(x) $64^{\frac{4}{3}}$

$= \left(\sqrt[3]{64}\right)^4$

$= 256$

Q. 8. (i) $100^{-\frac{1}{2}}$

$= \dfrac{1}{100^{\frac{1}{2}}}$

$= \dfrac{1}{\sqrt{100}}$

$= \dfrac{1}{10}$

(ii) $36^{-\frac{1}{2}}$

$= \dfrac{1}{36^{\frac{1}{2}}}$

$= \dfrac{1}{\sqrt{36}}$

$= \dfrac{1}{6}$

(iii) $16^{-\frac{1}{4}}$

$= \dfrac{1}{16^{\frac{1}{4}}}$

$= \dfrac{1}{\sqrt[4]{16}}$

$= \dfrac{1}{2}$

(iv) $9^{-\frac{3}{2}}$

$= \dfrac{1}{9^{\frac{3}{2}}}$

$= \dfrac{1}{\left(\sqrt{9}\right)^3}$

$= \dfrac{1}{27}$

(v) $81^{-\frac{3}{4}}$

$= \dfrac{1}{81^{\frac{3}{4}}}$

$= \dfrac{1}{\left(\sqrt[4]{81}\right)^3}$

$= \dfrac{1}{27}$

(vi) $8^{-\frac{2}{3}}$

$= \frac{1}{8^{\frac{2}{3}}}$

$= \frac{1}{\left(\sqrt[3]{8}\right)^2}$

$= \frac{1}{4}$

(vii) $9^{-\frac{5}{2}}$

$= \frac{1}{9^{\frac{5}{2}}}$

$= \frac{1}{\left(\sqrt{9}\right)^5}$

$= \frac{1}{243}$

(viii) $125^{-\frac{2}{3}}$

$= \frac{1}{125^{\frac{2}{3}}}$

$= \frac{1}{\left(\sqrt[3]{125}\right)^2}$

$= \frac{1}{25}$

(ix) $16^{-\frac{5}{4}}$

$= \frac{1}{16^{\frac{5}{4}}}$

$= \frac{1}{\left(\sqrt[4]{16}\right)^5}$

$= \frac{1}{32}$

(x) $100^{-\frac{5}{2}}$

$= \frac{1}{100^{\frac{5}{2}}}$

$= \frac{1}{\left(\sqrt{100}\right)^5}$

$= \frac{1}{100{,}000}$

Q. 9. (i) $20^4 = (5 \times 4)^4$

$= 5^4 \times 4^4$

(ii) $15^6 = (3 \times 5)^6$

$= 3^6 \times 5^6$

(iii) $36^{\frac{1}{2}} = (9 \times 4)^{\frac{1}{2}}$

$= 9^{\frac{1}{2}} \cdot 4^{\frac{1}{2}}$

(iv) $216^{\frac{1}{2}} = (8 \times 27)^{\frac{1}{3}}$

$= 8^{\frac{1}{3}} \cdot 27^{\frac{1}{3}}$

Q. 10. (i) $\frac{6^8}{8^8} = \left(\frac{6}{8}\right)^8$

$= \left(\frac{3}{4}\right)^8$

(ii) $\frac{9^9}{15^9} = \left(\frac{9}{15}\right)^9$

$= \left(\frac{3}{5}\right)^9$

(iii) $\frac{18^{\frac{1}{2}}}{32^{\frac{1}{2}}} = \left(\frac{18}{32}\right)^{\frac{1}{2}}$

$= \left(\frac{9}{16}\right)^{\frac{1}{2}}$

(iv) $\frac{75^{-\frac{1}{2}}}{192^{-\frac{1}{2}}} = \left(\frac{75}{192}\right)^{-\frac{1}{2}}$

$= \left(\frac{25}{64}\right)^{-\frac{1}{2}}$

Q. 11. (i) $\left(\frac{1}{4}\right)^{\frac{1}{2}}$

$= \frac{1}{\sqrt{4}}$

$= \frac{1}{2}$

(ii) $\left(\frac{1}{25}\right)^{\frac{1}{2}}$

$= \frac{1}{\sqrt{25}}$

$= \frac{1}{5}$

(iii) $\left(\frac{4}{9}\right)^{\frac{1}{2}}$

$= \frac{\sqrt{4}}{\sqrt{9}}$

$= \frac{2}{3}$

(iv) $\left(\frac{81}{25}\right)^{\frac{1}{2}}$

$= \frac{\sqrt{81}}{\sqrt{25}} = \frac{9}{5}$

(v) $\left(\frac{8}{27}\right)^{\frac{1}{3}}$

$= \frac{\sqrt[3]{8}}{\sqrt[3]{27}}$

$= \frac{2}{3}$

(vi) $\left(\frac{8}{125}\right)^{\frac{1}{3}}$

$= \frac{\sqrt[3]{8}}{\sqrt[3]{125}}$

$= \frac{2}{5}$

(vii) $\left(\frac{16}{81}\right)^{\frac{3}{4}}$

$= \frac{\left(\sqrt[4]{16}\right)^3}{\left(\sqrt[4]{81}\right)^3}$

$= \frac{8}{27}$

(viii) $\left(\frac{27}{64}\right)^{\frac{2}{3}}$

$= \frac{\left(\sqrt[3]{27}\right)^2}{\left(\sqrt[3]{64}\right)^2}$

$= \frac{9}{16}$

Q. 12. (i) $\left(\frac{36}{25}\right)^{-\frac{1}{2}}$

$= \frac{1}{\frac{\sqrt{36}}{\sqrt{25}}}$

$= \frac{5}{6}$

(ii) $\left(\frac{4}{121}\right)^{-\frac{1}{2}}$

$= \frac{1}{\frac{\sqrt{4}}{\sqrt{121}}}$

$= \frac{11}{2}$

(iii) $\left(\frac{8}{125}\right)^{-\frac{1}{3}}$

$= \frac{1}{\frac{\sqrt[3]{8}}{\sqrt[3]{125}}}$

$= \frac{5}{2}$

(iv) $\left(\frac{27}{1{,}000}\right)^{-\frac{2}{3}}$

$= \frac{1}{\frac{\left(\sqrt[3]{27}\right)^2}{\left(\sqrt[3]{1{,}000}\right)^2}}$

$= \frac{100}{9}$

(v) $\left(\frac{125}{27}\right)^{-\frac{2}{3}}$

$= \frac{1}{\frac{\left(\sqrt[3]{125}\right)^2}{\left(\sqrt[3]{27}\right)^2}}$

$= \frac{9}{25}$

(vi) $\left(\frac{4}{9}\right)^{-\frac{3}{2}}$

$= \frac{1}{\frac{(\sqrt{4})^3}{(\sqrt{9})^3}}$

$= \frac{27}{8}$

Q. 13.

(i) 4.36	(vi) 11.51
(ii) 57.19	(vii) 0.04
(iii) 1.9	(viii) 2.02
(iv) 15.61	(ix) 2.15
(v) 0.02	(x) 4.18

Q. 14. (i) $24 = 2 \times 2 \times 2 \times 3$

$= 2^3 \cdot 3$

(ii) $(24)^3 = (2^3 \cdot 3)^3$

$= (2^3)^3 \cdot 3^3$

$= 2^9 \cdot 3^3$

Q. 15. 28^{2012}

$28 = 2 \times 2 \times 7$

$28 = 2^2 \times 7$

$28^{2012} = (2^2 \cdot 7)^{2012}$

$= (2^2)^{2012} \cdot 7^{2012}$

$= 2^{4024} \cdot 7^{2012}$

Q. 16. $100 = 2 \times 2 \times 5 \times 5$

$= 2^2 \cdot 5^2$

$100 = 2^2 \cdot 5^2$

$100^{1601} = (2^2 \cdot 5^2)^{1601}$

$= (2^2)^{1601} \cdot (5^2)^{1601}$

$= 2^{3202} \cdot 5^{3202}$

Q. 17. (i) $144 = 12^2$

$12 = 2 \times 2 \times 3$

$12^2 = (2^2 \times 3)^2 = 2^4 \cdot 3^2$

$2{,}500 = 50^2$

$50 = 2 \times 5 \times 5$

$50 = 2 \times 5^2$

$50^2 = (2 \times 5^2)^2$

$= 2^2 \cdot 5^4$

$11{,}025 = 105^2$

$105 = 5 \times 3 \times 7$

$105^2 = (3 \times 5 \times 7)^2$

$105^2 = 3^2 \cdot 5^2 \cdot 7^2$

$1{,}002{,}001 = 1{,}001^2$

$1{,}001 = 7 \times 11 \times 13$

$(1{,}001)^2 = (7 \times 11 \times 13)^2$

$(1{,}001)^2 = 7^2 \cdot 11^2 \cdot 13^2$

(ii) even

(iii) $3 \times 23 = 69$

Reason: The prime factorisation of a square number must have prime factors with even powers.

Q. 18. (i) 2^2

(ii) 2^3

(iii) 2^4

(iv) 2^5

(v) 2^{-1}

(vi) 2^{-2}

(vii) $2^{\frac{1}{2}}$

(viii) $2^{\frac{1}{3}}$

(ix) $2^{-\frac{1}{2}}$

Q. 19. (i) 3^0

(ii) 3^2

(iii) 3^3

(iv) 3^4

(v) 3^{-1}

(vi) 3^{-2}

(vii) $3^{\frac{1}{2}}$

(viii) $3^{\frac{3}{2}}$

(ix) $3^{-\frac{1}{2}}$

Q. 20. (i) 5^2

(ii) 5^3

(iii) 5^{-1}

(iv) 5^0

(v) 5^{-2}

(vi) 5^{-3}

(vii) $5^{\frac{1}{2}}$

(viii) $5^{\frac{1}{5}}$

(ix) $5^{-\frac{1}{2}}$

(x) 5^{-1}

Q. 21. (i) 10^2 (iv) 10^4

(ii) 10^3 (v) 10^{-1}

(iii) 10^{-2} (vi) 10^{-4}

(vii) $10^{\frac{1}{2}}$

(viii) $10^{2.5}$

(ix) $10^{-\frac{1}{2}}$

(x) $10^{\frac{1}{6}}$

(xi) $10^{\frac{3}{2}}$

(xii) $10^{1.5}$

Exercise 9.3

Q. 1. (i) $2^x = 4$

$2^x = 2^2$

$x = 2$

(ii) $3^x = 27$

$3^x = 3^3$

$x = 3$

(iii) $5^x = 125$

$5^x = 5^3$

$x = 3$

(iv) $10^x = 1{,}000$

$10^x = 10^3$

$x = 3$

(v) $4^x = 64$

$4^x = 4^3$

$x = 3$

(vi) $3^x = 81$

$3^x = 3^4$

$x = 4$

(vii) $10^x = 10{,}000$

$10^x = 10^4$

$x = 4$

(viii) $6^x = 216$

$6^x = 6^3$

$x = 3$

(ix) $7^x = 49$

$7^x = 7^2$

$x = 2$

(x) $3^x = 729$

$3^x = 3^6$

$x = 6$

Q. 2. (i) $9^x = 3^4$

$(3^2)^x = 3^4$

$3^{2x} = 3^4$

$2x = 4$

$x = 2$

(ii) $4^x = 8^2$

$(2^2)^x = (2^3)^2$

$2^{2x} = 2^6$

$2x = 6$

$x = 3$

(iii) $5^x = 25^2$

$5^x = (5^2)^2$

$5^x = 5^4$

$x = 4$

(iv) $10^x = 100^3$

$10^x = (10^2)^3$

$10^x = 10^6$

$x = 6$

(v) $11^x = 121^5$

$11^x = (11^2)^5$

$11^x = 11^{10}$

$x = 10$

(vi) $2^x = 16^5$

$2^x = (2^4)^5$

$2^x = 2^{20}$

$x = 20$

(vii) $4^{2x} = 8^3$

$(2^2)^{2x} = (2^3)^3$

$2^{4x} = 2^9$

$4x = 9$

$x = \frac{9}{4}$

(viii) $3^{3x} = 27^2$

$3^{3x} = (3^3)^2$

$3x = 6$

$x = 2$

(ix) $4^{5x} = 8^5$

$(2^2)^{5x} = (2^3)^5$

$2^{10x} = 2^{15}$

$10x = 15$

$x = \frac{3}{2}$

(x) $a^{2x} = (a^2)^3$

$a^{2x} = a^6$

$2x = 6$

$x = 3$

Q. 3. (i) $2^x = 2^7\sqrt{2}$

$2^x = 2^7 \cdot 2^{\frac{1}{2}}$

$2^x = 2^{7\frac{1}{2}}$

$x = 7\frac{1}{2}$

(ii) $2^x = \frac{2^7}{4}$

$2^x = \frac{2^7}{2^2}$

$2^x = 2^5$

$x = 5$

(iii) $5^x = \frac{125}{\sqrt{5}}$

$5^x = \frac{5^3}{5^{\frac{1}{2}}}$

$5^x = 5^{2.5}$

$x = 2.5$

(iv) $3^{x+1} = \frac{9}{\sqrt{3}}$

$3^{x+1} = \frac{3^2}{3^{\frac{1}{2}}}$

$3^{x+1} = 3^{1\frac{1}{2}}$

$x + 1 = 1\frac{1}{2}$

$x = \frac{1}{2}$

(v) $10^{x-3} = \frac{\sqrt{10}}{100}$

$10^{x-3} = \frac{10^{\frac{1}{2}}}{10^2}$

$10^{x-3} = 10^{-1.5}$

$x = 1.5$

(vi) $7^x = 7^{1\frac{2}{3}}$

$x = 1\frac{2}{3}$

(vii) $10^{2x-1} = \frac{(10^3)^{\frac{1}{2}}}{10}$

$10^{2x-1} = \frac{10^{\frac{3}{2}}}{10^1}$

$10^{2x-1} = 10^{\frac{1}{2}}$

$2x - 1 = \frac{1}{2}$

$2x = 1\frac{1}{2}$

$x = \frac{3}{4}$

(viii) $4^x = \frac{32\sqrt{2}}{2}$

$2^{2x} = \frac{2^5 \cdot 2^{\frac{1}{2}}}{2}$

$2^{2x} = 2^{4.5}$

$2x = 4.5$

$x = 2\frac{1}{4}$

(ix) $49^x = \frac{49}{\sqrt{7}}$

$7^{2x} = \frac{7^2}{7^{\frac{1}{2}}}$

$7^{2x} = 7^{1.5}$

$2x = 1.5$

$x = \frac{3}{4}$

(x) $36^{x+2} = \frac{216}{\sqrt{6}}$

$(6^2)^{x+2} = \frac{6^3}{6^{\frac{1}{2}}}$

$6^{2x+4} = 6^{2.5}$

$2x + 4 = 2.5$

$2x = -1.5$

$x = -\frac{3}{4}$

Q. 4. (i) 2^4

(ii) 2^3

(iii) $2^{\frac{3}{2}}$

(iv) $\dfrac{16}{\sqrt{8}} = \dfrac{2^4}{2^{\frac{3}{2}}}$

$= 2^{2.5}$

$2^{2x-1} = \left(\dfrac{16}{\sqrt{8}}\right)^3$

$2^{2x-1} = (2^{2.5})^3$

$2x - 1 = 7.5$

$2x = 8.5$

$x = 4.25$

Q. 5. $\dfrac{9}{3^{1-n}} = 81$

$\dfrac{3^2}{3^{1-n}} = 3^4$

$3^{1+n} = 3^4$

$n = 3$

Q. 6. $\dfrac{125 \times 5^{2n}}{5^{n+1}} = 625$

$\dfrac{5^3 \times 5^{2n}}{5^{n+1}} = 5^4$

$5^{3+2n-n-1} = 5^4$

$2 + n = 4$

$n = 2$

Q. 7. $\dfrac{7^2 \times 49^x}{7^{1+x}} = 343$

$\dfrac{7^2 \times 7^{2x}}{7^{1+x}} = 7^3$

$7^{2+2x-1-x} = 7^3$

$x + 1 = 3$

$x = 4$

Q. 8. $p - q = 96$

$p + q = 160$

$2p = 256$ $\qquad$ $q = 160 - p$

$p = 128$ $\qquad$ $q = 32$

$2^x = p$ $\qquad$ $2^y = q$

$2^x = 128$ $\qquad$ $2^y = 32$

$2^x = 2^7$ $\qquad$ $2^y = 2^5$

$x = 7$ $\qquad$ $y = 5$

Q. 9. Let $3^x = m$ and $3^y = n$

$5m - 2n = 387$

$\ominus 5m \oplus 4n = \ominus 369$

$2n = 18$

$n = 9$

$3^y = 9$

$\therefore y = 2$

$x = 4$ and $y = 2$

$5m = 387 + 2(9)$

$5m = 405$

$m = 81$

$3^x = 81$

$\therefore x = 4$

Q. 10. $\sqrt{y-5} = 5 \Rightarrow y = 30$

$2^x = 8 \Rightarrow x = 3$

$x + y = 30 + 3$

$= 33$

Q. 11. $6^7 + 4^5 = 280{,}960$

Q. 12. $\sqrt{244 + \sqrt{144}} = \sqrt{244 + 12} = \sqrt{256} = 16 = 2^4$

Q. 13.

$$\begin{array}{r|l} 3 & 75 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$75 = 3 \times 5^2$

$\frac{5^x}{3} = \frac{5^6}{3 \times 5^2}$

$5^2 \cdot 5^x = 5^6$

$5^{2+x} = 5^6$

$\therefore 2 + x = 6$

$x = 4$

Q. 14. (i) $\left(8^{\frac{1}{3}}\right)\left(4^{\frac{1}{2}}\right) = 2^{5-x}$

$(2)(2) = 2^{5-x}$

$2^2 = 2^{5-x}$

$\therefore 5 - x = 2$

$x = 3$

(ii) $\left(27^{\frac{1}{3}}\right)\left(9^{\frac{1}{2}}\right) = 3^{5-x}$

$(3)(3) = 3^{5-x}$

$3^2 = 3^{5-x}$

$\therefore 5 - x = 2$

$x = 3$

Q. 15. $16 = 2^4$, $2\sqrt{2} = 2^{1\frac{1}{2}}$

$2^{2x-1} = \left(\frac{2^4}{2^{1\frac{1}{2}}}\right)^3$

$2^{2x-1} = \left(2^{2\frac{1}{2}}\right)^3$

$2^{2x-1} = 2^{7\frac{1}{2}}$

$\therefore 2x - 1 = 7\frac{1}{2}$

$2x = 8\frac{1}{2}$

$x = 4\frac{1}{4}$

Q. 16.

$2^2 - 2$	$2^3 - 2^2$	$2^4 - 2^3$	$2^5 - 2^4$	$2^6 - 2^5$	$2^7 - 2^6$
2	2^2	2^3	2^4	2^5	2^6

$2^{p+1} - 2^p = 2^p$

Q. 17. $\frac{2^{12}}{2^4} = 2^x$

$2^x = 2^8$

$\therefore x = 8$

Q. 18. $\frac{2^{16}}{2^3} = 2^{y-1}$

$2^{y-1} = 2^{13}$

$\therefore y - 1 = 13$

$y = 14$

Exercise 9.4

Q. 1. (i) 5.3×10^3

(ii) 1.75×10^5

(iii) 2.4×10^4

(iv) 2.35×10^8

(v) 7.376×10^6

(vi) 2.0×10^{-2}

(vii) 1.5×10^{-3}

(viii) 1.67×10^{-4}

(ix) 6.12×10^{-3}

(x) 2.3×10^{-5}

Q. 2. (i) 400,000

(ii) 16,000,000

(iii) 5,400

(iv) 8,200,000

(v) 940

(vi) 0.0019

(vii) 0.000236

(viii) 0.026

(ix) 0.56

(x) 0.00000506

Q. 3. (i) 6.2×10^3

(ii) $520 \times 10^2 - 3.8 \times 10^2 = 516.2 \times 10^2$

$= 5.162 \times 10^4$

(iii) $14.688 \times 10^8 = 1.4688 \times 10^9$

(iv) 1.344×10^{11}

Q. 4. (i) 3.6×10^{-5}

(ii) 5.613×10^{-4}

(iii) 3.45×10^{-2}

(iv) 6.3×10^{-4}

(v) 7.8×10^{-3}

(vi) 4.04×10^{-2}

Q. 5. (i) 4

(ii) 5

(iii) 3

(iv) 3

(v) 2

(vi) 3

Q. 6. Distance = speed × time

$= (2.9 \times 10^8)(480)$

$= 1.392 \times 10^{11}$ m

$= 1.392 \times 10^8$ km

Q. 7. 10^{10} angstroms = 1 m

1 angstrom $= \frac{1}{10^{10}}$ m

$= 10^{-10}$ m

1.6 angstroms $= 1.6 \times 10^{-10}$ m

Q. 8. (i) 3.66×10^4 m

(ii) 3.68×10^7 m^2

Q. 9. $\frac{1.36 \times 10^8}{32{,}000} = 4.25 \times 10^3$ images

Q. 10. $3.8 \times 10^3 \times 2.6 \times 10^2 = 988{,}000$ litres

Q. 11. (i) 140 million $= 1.4 \times 10^8$

(ii) $\frac{1.4 \times 10^8}{72} = 1.94 \times 10^6$

Q. 12. (i) $5.6 \times 10^8 + 7.4 \times 10^8 = 13 \times 10^8$

$= 1.3 \times 10^9$

(ii) $\frac{1.3 \times 10^9}{6.7 \times 10^9} \times 100 = 19.4\%$

Q. 13. (i) Difference $= 1.21 \times 10^5 - 1.21 \times 10^4$

$= 108{,}900$ km

(ii) Equator (Venus) $= 2\pi\,(1.21 \times 10^4)$

Equator (Saturn) $= 2\pi\,(1.21 \times 10^5)$

Difference $= 2\pi\,[1.21 \times 10^5 - 1.21 \times 10^4] \approx 6.8 \times 10^5$

$= 6$ orders of magnitude

(iii) Yes, it is 2π times the difference in orders of magnitude between both radii.

Q. 14. Width of gold leaf $= \frac{1}{10{,}000{,}000} = 1.0 \times 10^{-8}$ m

Width of an atom of gold $= 0.26 \times 10^{-9}$ m

Number of atoms thick $= \dfrac{1.0 \times 10^{-8}}{0.26 \times 10^{-9}}$

≈ 38 atoms

Q. 15. (i) 1.0×10^{-9} seconds

(ii) $\dfrac{1}{3(1.0 \times 10^{-9})} = 333{,}333{,}333$ calculations

Q. 16. (i) Jupiter

(ii) Saturn

(iii) 1.906×10^{27} kg

(iv) 1.894×10^{27} kg

(v) $5 - 4 = 1$ order of magnitude

Revision Exercises

Q. 1. (i) 7^{11}

(ii) $\dfrac{(2^2)^5}{2^3} = \dfrac{2^{10}}{2^3}$

$= 2^7$

(iii) 2^{10}

(iv) 11^7

Q. 2. (i) 5^{-4}

(ii) $12^{\frac{1}{6}}$

(iii) $17^{\frac{3}{5}}$

(iv) $\dfrac{4^2}{4^{\frac{5}{2}}} = 4^{-\frac{1}{2}}$

Q. 3. (i) $(6^{10})^2 = 6^{20}$

(ii) $(8^{11})^2 = 8^{22}$

(iii) $(2^4)^6 = 2^{24}$

(iv) $(4^5)^7 = 4^{35}$

Q. 4. (i) p^9

(ii) p^{20}

(iii) $p^{\frac{6}{4}} = p^{\frac{3}{2}}$

(iv) $(p^{16})^{\frac{1}{5}} = p^{\frac{16}{5}}$

Q. 5. (i) 5

(ii) $9 = 3^2$, $\sqrt{3} = 3^{\frac{1}{2}}$

$3^x = \dfrac{3^2}{3^{\frac{1}{2}}}$

$3^x = 3^{1\frac{1}{2}}$

$\therefore x = 1\frac{1}{2}$

Q. 6. $\dfrac{(2^2)^{2012} \times 3^{2011}}{(3 \times 2)^{2011} \times 2^{2011}}$

$= \dfrac{2^{4024} \times 3^{2011}}{3^{2011} \times 2^{2011} \times 2^{2011}}$

$= \dfrac{2^{4024}}{2^{4022}}$

$= 2^2$

Q. 7. (i) $64^{-\frac{1}{2}} = \dfrac{1}{64^{\frac{1}{2}}} = \dfrac{1}{8}$

(ii) $128 = 2^7$, $\sqrt{2} = 2^{\frac{1}{2}}$

$2^{2x+1} = \dfrac{2^7}{2^{\frac{1}{2}}}$

$2^{2x+1} = 2^{6\frac{1}{2}}$

$\therefore 2x + 1 = 6\frac{1}{2}$

$2x = 5\frac{1}{2}$

$x = \dfrac{11}{4}$

Q. 8. $x + y = 0$

$\therefore y = -x$

$$\frac{x^{2012}}{y^{2012}} + 100 = \frac{x^{2012}}{(-x)^{2012}} + 100$$

$$= \frac{x^{2012}}{x^{2012}} + 100$$

$$= 1 + 100$$

$$= 101$$

Q. 9. $5x + 4y = 5 \times 4 \times 10^{-5} + 4 \times 6 \times 10^{-7}$

$= 20 \times 10^{-5} + 24 \times 10^{-7}$

$= 2000 \times 10^{-7} + 24 \times 10^{-7}$

$= 2024 \times 10^{-7}$

$= 2.024 \times 10^{-4}$

Q. 10. $\text{googol}^2 = (10^{100})^2 = 10^{200}$

(i) $100 \times \text{googol} = 100 \times 10^{100} = 10^{102}$

$3 \times \text{googol} = 3 \times 10^{100}$

$\text{googol} = 10^{100}$

$\text{googol}^3 = (10^{100})^3 = 10^{300}$

In order of increasing magnitude

googol, $3 \times$ googol, $100 \times$ googol, googol^2, googol^3

(ii) googol^3

(iii) 100 orders of magnitude

(iv) googol^2 and googol^3

Q. 11. $n^2 - 5n + 5 = 1$

$\therefore n^2 - 5n + 4 = 0$

$(n - 4)(n - 1) = 0$

$n = 1$ or $n = 4$

$n^2 + 2n - 24 = 0$

$(n + 6)(n - 4) = 0$

$n = -6$ or $n = 4$

$n^2 - 5n + 5 = -1$

$n^2 - 5n + 6 = 0$

$(n - 3)(n - 2) = 0$

$n = 3$ or $n = -2$

If $n = 3$, then

$n^2 + 2n - 24$ is odd

Therefore, $n = 3$ is not a solution.

If $n = -2$

$n^2 + 2n - 24$ is even

Therefore, $n = -2$ is a solution.

Solutions: $n = -6$ or $n = -2$ or $n = 1$ or $n = 4$

Q. 12. (i) $a^4 + a^4 + a^4 + a^4 = 4a^4$

$2^4 + 2^4 + 2^4 + 2^4 = 4(2^4)$

$= 2^2\,(2^4) = 2^6$

(ii) $2^{\frac{1}{4}} + 2^{\frac{1}{4}} + 2^{\frac{1}{4}} + 2^{\frac{1}{4}} = 4\left(2^{\frac{1}{4}}\right)$

$= 2^2\left(2^{\frac{1}{4}}\right)$

$= 2^{\frac{9}{4}}$

(iii) $3a^5 + 6a^5 = 9a^5$

$3(3^5) + 6(3^5) = 9(3^5) = 3^2\,(3^5) = 3^7$

(iv) $27(3^8) = 3^3\,(3^8) = 3^{11}$

Q. 13. $3 \times 10^4 + 5 \times 10^4 = 8 \times 10^4$

Joe is correct.

Q. 14. $16^{-\frac{3}{4}} = \dfrac{1}{16^{\frac{3}{4}}} = \dfrac{1}{\left(16^{\frac{1}{4}}\right)^3} = \dfrac{1}{2^3} = \dfrac{1}{8}$

$64^{-\frac{1}{2}} = \dfrac{1}{64^{\frac{1}{2}}} = \dfrac{1}{8}$

$8^{-\frac{2}{3}} = \dfrac{1}{8^{\frac{2}{3}}} = \dfrac{1}{\left(8^{\frac{1}{3}}\right)^2} = \dfrac{1}{4}$

$\therefore\ 8^{-\frac{2}{3}}$ is the odd one out.

Q. 15. Mach 3 = 3708 km/hr

= 61.8 km/min

In 5 minutes the plane would travel $61.8 \times 5 = 309$ km.

Q. 16. (i) % through Dublin: $18.7/22.7 \times 100 = 8.24 \times 10^1$ %

(ii) 15.6×18.7 million = 291,720,000 kg

(iii) 97% = 2,400,000

100% = 2.474×10^6

Q. 17. 3.9×10^{20} is 20 orders of magnitude

5.9×10^{12} is 13 orders of magnitude

Therefore, the radii differ by 7 orders of magnitude.

Q. 18. Number of light years: $4.047 \times 10^{13} \div 9.5 \times 10^{12}$

= 4.26 light years

Q. 19. $\dfrac{803{,}000}{0.00000525} = 1.53 \times 10^{11}$ bits/sec