Chapter 14 Exercise 14.1

Q. 1. $x + 2y = 3z$

$x + 2y - 2y = 3z - 2y$

$\Rightarrow x = 3z - 2y$

Q. 2. $p + 4q = 2$

$p + 4q - 4q = 2 - 4q$

$\Rightarrow p = 2 - 4q$

Q. 3. $2b - 5 = a$

$2b = a + 5$

$\frac{2b}{2} = \frac{a + 5}{2}$

$\Rightarrow b = \frac{(a + 5)}{2}$

Q. 4. $t = a + b + c$

$t - b - c = a + b + c - b - c$

$\Rightarrow a = t - b - c$

Q. 5. $l + 2 = xy$

$\frac{l + 2}{x} = \frac{xy}{x}$

$\Rightarrow y = \frac{l + 2}{x}$

Q. 6. $r - 4u = pt$

$-4u = pt - r$

$u = \frac{pt - r}{-4} = \frac{-(pt - r)}{4}$

$\Rightarrow u = \frac{r - pt}{4}$

Q. 7. $a - b = at$

$a - at = b$

$a(1 - t) = b$

$\Rightarrow a = \frac{b}{1 - t}$

Q. 8. $x = 2yz - z$

$x = z(2y - 1)$

$\Rightarrow z = \frac{x}{2y - 1}$

Q. 9. $3a = \frac{b + c}{2}$

$6a = b + c \Rightarrow b = 6a - c$

Q. 10. $p = \frac{q - 3r}{4}$

$4p = q - 3r$

$4p - q = -3r$

$3r = q - 4p$

$\Rightarrow r = \frac{q - 4p}{3}$

Q. 11. $\frac{a}{2} = \frac{4b + c}{3}$

$6\left(\frac{a}{2}\right) = 6\left(\frac{4b + c}{3}\right)$

$3a = 2(4b + c)$

$3a = 8b + 2c$

$8b = 3a - 2c$

$\Rightarrow b = \frac{3a - 2c}{8}$

Q. 12. $\frac{p}{2} - \frac{3q}{5} = \frac{c}{10}$

$10\left(\frac{p}{2}\right) - 10\left(\frac{3q}{5}\right) = 10\left(\frac{c}{10}\right)$

$5p - 6q = c$

$c = 5p - 6q$

Q. 13. $A = \frac{1}{2}(a + b)h$

$2A = 2\left(\frac{1}{2}\right)(a + b)h$

$2A = (a + b)h$

$\frac{2A}{h} = a + b$

$\Rightarrow a = \frac{2A}{h} - b$ OR $\frac{2A - bh}{h}$

Q. 14. $r - 2rs = q$

$r(1 - 2s) = q$

$\Rightarrow r = \frac{q}{1 - 2s}$

Q. 15. $pq = p - r$

$r = p - pq$

$r = p(1 - q)$

$\Rightarrow p = \frac{r}{1 - q}$

Q. 16. $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$

$uvf\left(\frac{1}{u}\right)+uvf\left(\frac{1}{v}\right)=uvf\left(\frac{1}{f}\right)$

$vf+uf=uv$

$f(u+v)=uv$

$f=\frac{uv}{u+v}$

Q. 17. $\frac{2}{a}-\frac{3}{b}=\frac{1}{c}$

$abc\left(\frac{2}{a}\right)-abc\left(\frac{3}{b}\right)=abc\left(\frac{1}{c}\right)$

$2bc-3ac=ab$

$2b-ab=3ac$

$b(2-a)=3ac$

$\Rightarrow b=\frac{3ac}{2-a}$

Q. 18. $p=c+pq$

$p-pq=c$

$p(1-q)=c$

$\Rightarrow p=\frac{c}{1-q}$

Q. 19. $a=\frac{bc-d}{3c}$

$3ac=bc-d$

$3ac-bc=-d$

$c(3a-b)=-d$

$c=\frac{-d}{3a-b}$

$\Rightarrow c=\frac{d}{b-3a}$

Q. 20. $a=\frac{p}{p+b}$

$a(p+b)=p$

$ap+ab=p$

$ab=p-ap$

$ab=p(1-a)$

$\Rightarrow p=\frac{ab}{1-a}$

Exercise 14.2

Q. 1. $c=\sqrt{a^2+b^2}$

$c^2=(\sqrt{a^2+b^2})^2$

$c^2=a^2+b^2$

$a^2=c^2-b^2 \Rightarrow a=\pm\sqrt{c^2-b^2}$

Q. 2. $3a^2-2b=c$

$3a^2=2b+c$

$a^2=\frac{2b+c}{3}$

$a=\pm\sqrt{\frac{2b+c}{3}}$

Q. 3. $r=\sqrt{pq+q}$

$r^2=pq+q$

$r^2=q(p+1)$

$\Rightarrow q=\frac{r^2}{p+1}$

Q. 4 $\sqrt{\frac{a+b}{2}}=3$

$\left(\sqrt{\frac{a+b}{2}}\right)^2=3^2$

$\frac{a+b}{2}=9$

$a+b=18$

$b=18-a$

Q. 5. $\sqrt{\frac{p}{r-q}}=p$

$\frac{p}{r-q}=p^2$

$p=p^2(r-q)$

$\frac{p}{p^2}=r-q$

$\frac{1}{p}=r-q$

$\Rightarrow r=\frac{1}{p}+q$

Q. 6. $T = 2\pi\sqrt{\frac{l}{g}}$

$$T^2 = \left(2\pi\sqrt{\frac{l}{g}}\right)^2$$

$$T^2 = (2)^2(\pi)^2\left[\frac{(\sqrt{l})^2}{(\sqrt{g})^2}\right]$$

$$T^2 = 4\pi^2\frac{l}{g}$$

$$T^2(g) = (g)4\pi^2\frac{l}{g}$$

$$T^2 g = 4\pi^2 l$$

$$g = \frac{4\pi^2 l}{T^2}$$

Q. 7. $\frac{x}{b} = \frac{b^2}{a-d}$

$$(b)(a-d)\frac{x}{b} = (b)(a-d)\frac{b^2}{a-d}$$

$$(a-d)x = (b)b^2$$

$$ax - dx = b^3$$

$$\Rightarrow b = \sqrt[3]{ax - dx}$$

Q. 8. $a = \frac{3\sqrt{rt}}{2}$

$$a^2 = \left(\frac{3\sqrt{rt}}{2}\right)^2 \Rightarrow a^2 = \frac{9rt}{4}$$

$$4a^2 = 9rt$$

$$\frac{4a^2}{9r} = t$$

$$t = \frac{4a^2}{9r}$$

Q. 9. (i) $y = ax - 1$

$$y = a(2a-1) - 1$$

$$y = 2a^2 - a - 1$$

(ii) $y = (2a+1)(a-1)$

Q. 10. (i) $x = ak + a^3$

$$x = a(4 - 2a^2) + a^3$$

$$x = 4a - 2a^3 + a^3$$

$$x = 4a - a^3$$

(ii) $x = a(4 - a^2)$

$$= a(2-a)(2+a)$$

(iii) $a = 2$

$$x = 2(2-2)(2+2)$$

$$= 2(0)(4) = 0$$

Q. 11. (i) $a + y = 2x$

$$\Rightarrow a = 2x - y$$

$$2b - 2y = 5x$$

$$(\times 5) \quad 10b - 10y = 25x$$

$$\Rightarrow 10b = 25x + 10y$$

$$\Rightarrow a + 10b = 2x - y + (25x + 10y)$$

$$a + 10b = 27x + 9y$$

(ii) $a + 10b = 9(3x + y)$

$$a + 10b = 9(4)$$

$$a + 10b = 36$$

Q. 12. (i) $V = \left(\frac{1}{3}\right)\pi r^2 h$

$$V = \left(\frac{1}{3}\right)(3.14)(2)^2(10)$$

$$= \frac{(3.14)(4)(10)}{3}\text{ cm}^3$$

$$V = 41\frac{13}{15} \quad (41.87)\text{ cm}^3$$

(ii) $V = \frac{1}{3}\pi r^2 h$

$$3V = \pi r^2 h$$

$$\Rightarrow h = \frac{3V}{\pi r^2}$$

$$h = \frac{(3)(2{,}143.68)}{\left(\frac{22}{7}\right)(12^2)}$$

$$= \frac{(3)(2{,}143.68)}{\left(\frac{22}{7}\right)(144)}$$

$$= \frac{6{,}431.04}{452.57} = 14.21\text{ mm}$$

(iii) $V = \frac{1}{3}\pi r^2 h$

$$3V = \pi r^2 h$$

$$r^2 = \frac{3V}{\pi h}$$

$$\Rightarrow r = \sqrt{\frac{3V}{\pi h}}$$

$$r = \sqrt{\frac{(3)(301.44)}{(3.14)(8)}}$$

$$r = \sqrt{\frac{904.32}{25.12}}$$

$$r = \sqrt{36} \Rightarrow r = 6\text{ cm}$$

Q. 13. (i) $R = \sqrt{\frac{-4ac}{2a}}$

$2aR = \sqrt{-4ac}$

$(2aR)^2 = (\sqrt{-4ac})^2$

$4R^2a^2 = -4ac$

$c = \frac{4R^2a^2}{-4a}$

$\Rightarrow c = -R^2a$

(ii) $c = -(\sqrt{5})^2(13)$

$c = -(5)(13)$

$\Rightarrow c = -65$

Q. 14. (i) $v = \sqrt{2gh}$

$= \sqrt{2(9.8)(40)}$

$v = \sqrt{784}$

$= 28$ m/s

(ii) $v = \sqrt{2gh}$

$v^2 = 2gh$

$\Rightarrow h = \frac{v^2}{2g}$

$h = \frac{(42)^2}{2(9.8)}$

$= \frac{1764}{19.6} = 90$ metres

(iii) $v = \sqrt{2gh}$

$v^2 = 2gh$

$\Rightarrow g = \frac{v^2}{2h}$

$g = \frac{(78)^2}{2(117)}$

$g = \frac{6{,}084}{234}$

$\Rightarrow g = 26$ m/s^2

Q. 15. (a) $E = \left(\frac{1}{2}\right)mv^2$

(i) $E = \left(\frac{1}{2}\right)(10)(5^2) = 125$ Joules

(ii) $E = \left(\frac{1}{2}\right)(5)(7^2) = 122.5$ Joules

$\Rightarrow$ (i) Object of mass 10 kg travelling at 5 m/s has greater kinetic energy.

(b) $E = \left(\frac{1}{2}\right)(m)(v^2)$

$120.05 = \left(\frac{1}{2}\right)m(4.9)^2$

$m = \frac{2(120.05)}{(4.9)^2}$

$m = \frac{240.1}{24.01}$

$\Rightarrow m = 10$ kg

(c) (i) $E = \left(\frac{1}{2}\right)mv^2$

$2E = mv^2$

$v^2 = \frac{2E}{m}$

$\Rightarrow v = \sqrt{\frac{2E}{m}}$

(ii) $v = \sqrt{\frac{2(500)}{2}}$

$v = \sqrt{500}$

$\Rightarrow v = 22.36$ m/s

Q. 16. (a) $s = ut + \left(\frac{1}{2}\right)at^2$

$ut = s - \frac{1}{2}at^2$

$u = \frac{s}{t} - \frac{1}{2}at$

(b) $s = ut + \left(\frac{1}{2}\right)at^2$

$\frac{1}{2}at^2 = s - ut$

$at^2 = 2(s - ut)$

$a = \frac{2(s - ut)}{t^2}$ OR $a = \left[\frac{2s}{t^2} - \frac{u}{t}\right]$

(c) $s = 9$

$t = 3$

$u = 0$

$a = ?$

$a = \frac{2(9 - (0)(3))}{(3)^2}$

$a = \frac{18}{9}$

$= 2$ m/s^2

Q. 17. (a) $\frac{100A}{P} = 100 + r$

$\Rightarrow r = 100\left(\frac{A}{P}\right) - 100$

$r = 100\left(\frac{A}{P} - 1\right)$

(b) $r = 100\left(\frac{3{,}727.15}{3{,}500} - 1\right)$

$= 100(1.0649 - 1)$

$= 100(0.0649)$

$r = 6.49\%$

(c) $A = P(1 + i)^T$

$A = P(1 + i)^2$

(d) $A = P(1 + i)^2$

$\frac{A}{P} = (1 + i)^2$

$\sqrt{\frac{A}{P}} = \sqrt{(1 + i)^2}$

$1 + i = \sqrt{\frac{A}{P}}$

$\Rightarrow i = \sqrt{\frac{A}{P}} - 1$

(e) $i = \sqrt{\frac{40{,}470.91}{38{,}000.000}} - 1$

$i = \sqrt{1.065} - 1$

$i = 1.032 - 1$

$i = 0.032$

$i = 3.2\%$

Q. 18. (a) $c = \frac{5}{9}(F - 32)$

$c = \frac{5}{9}(77 - 32)$ San Francisco

$c = \frac{5}{9}(45)$

$c = 25$

Temperature in San Francisco $= 25°C$

Temperature in Dublin $= 24°C$

$\Rightarrow$ San Francisco is predicted to have a higher temperature.

(b) $C = \frac{5}{9}(F - 32)$

$9C = 5(F - 32)$

$\frac{9}{5}C = F - 32$

$\Rightarrow F = \frac{9}{5}C + 32$

(c) $K = C + 273.15$

$0 = C + 273.15$

$\Rightarrow C = -273.15$

$F = \frac{9}{5}C + 32$

$F = \frac{9}{5}(-273.15) + 32$

$F = -491.67 + 32$

$F = -459.67$ (degrees Fahrenheit)

Q. 19. (a) $C = \left(\frac{30}{68}\right) \times 125$ mg

$C = 55$ mg (to the nearest 5 mg)

(b) Let: C = Child's dose
A = Adult dose
x = Child's age
} for (i), (ii) and (iii)

(i) $C = \left(\frac{x}{x + 12}\right) \times A$

(ii) $C = \frac{A}{\left(\frac{x + 12}{x}\right)}$

(iii) $C = \frac{Ax}{x + 12}$

(c) By algebraic manipulation each formula can be written in the same form $C = \frac{xA}{x + 12}$:

Part (i) $C = \frac{xA}{x + 12}$

Part (ii) $C = \frac{A}{1} \times \left(\frac{x}{x + 12}\right)$

$= \frac{Ax}{x + 12} = \frac{xA}{x + 12}$

Part (iii) $C = \frac{xA}{x + 12}$

as $\frac{xA}{x + 12} = \frac{Ax}{x + 12}$

Hence, all 3 formulae give the same result.

(d) $C = \frac{Ax}{x + 12}$

$C = \frac{(150)(6)}{6 + 12} = \frac{900}{18}$

$C = 50$ mg per day

(e) $C = \frac{Ax}{x + 12}$

$\frac{C}{A} = \frac{x}{x + 12}$ but $\frac{C}{A} = \frac{1}{5}$

$\frac{1}{5} = \frac{x}{x + 12}$

$x + 12 = 5x$

$\Rightarrow 4x = 12$

$\Rightarrow x = \frac{12}{4} = 3$

Child's age is 3 years old